1. ProtunerPlus™ Model Identification

The ProtunerPlus™ Loop Analysis routine performs a Digital Laplace Transformation of the user identified test data. The solution for Dynamic Gain and Phase Shift at all frequencies is presented in the Bode, Nichols, and Nyquist Plots. The Frequency Plots are then used determine the optimum tuning parameters using curve fit technology to reshape the plots.

1.1 Digital Laplace Transformation

The ProtunerPlus™ Loop Analysis Procedure allows the user to determine the best response of the process variable to a step change in the controller output from which to find the optimum tuning parameters for the controller. Real processes are only step-wise linear with motion non-linearity such as dead band and hysteresis in the valve with non-linear installed characteristics. The ProtunerPlus™ allows the user to "window" the test data that best identifies the process response that the user determines will provide the best and most robust tuning. When the real process variable contains excessive noise in the measurement the ProtunerPlus™ Data Edit function allows the user to draw through the noise to best describe the real response of the process variable and thus the true transfer function of the process upon which to base the tuning. The controller filter time constant is found by the user to minimise the noise on the measurement. The ProtunerPlus™ Loop Analysis then determines the optimum tuning for the controller with the controller filter time constant determined by the user. The ProtunerPlus™ was developed for testing and tuning real world processes and these features and functions give the user the tools to determine the process transfer function not only from perfect data but test data from real process in the real world.

At this point let's discuss the methodology employed by the ProtunerPlus™ Loop Analysis to determine the loop transfer function.

Equation 1-1 Laplace Transformation of the Time Domain Equation

$$F(s) = L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$$

The Laplace transfer function of a process is the Laplace transformation of the input X(s) (controller output) divided by the Laplace transformation of the output Y(s) (process variable response)

Equation 1-2 Laplace Transformation of Process

$$P(s) = \frac{\int_0^\infty e^{-st} Y(t)dt}{\int_0^\infty e^{-st} X(t)dt}$$

Or

$$P(s) = \frac{Y(s)}{X(s)}$$

The time domain equations in Equation 1-1 {f(t)} and Equation 1-2 {Y(t) and X(t)} must be an equation from time zero to time infinity so that the time domain equation can be converted into the Laplace domain. In step testing a real process we only have data from the beginning of the test to the end of the test and not to infinity to describe both X(t) and Y(t).

Figure 1-1 is a time domain plot of a Multi-Order process with a 5th order plus deadtime transfer function with the following Laplace equation:

$$P(s) = \frac{1e^{-10s}}{(20s+1)(10s+1)(5s+1)(2.5s+1(1.25s+1)}$$

Figure 1-1 ProtunerPlus™ Open Loop Step Test on Multi-Order Process

The test data goes from time 0 to 300 seconds and from the **Size** display the graphs have 3001 points.

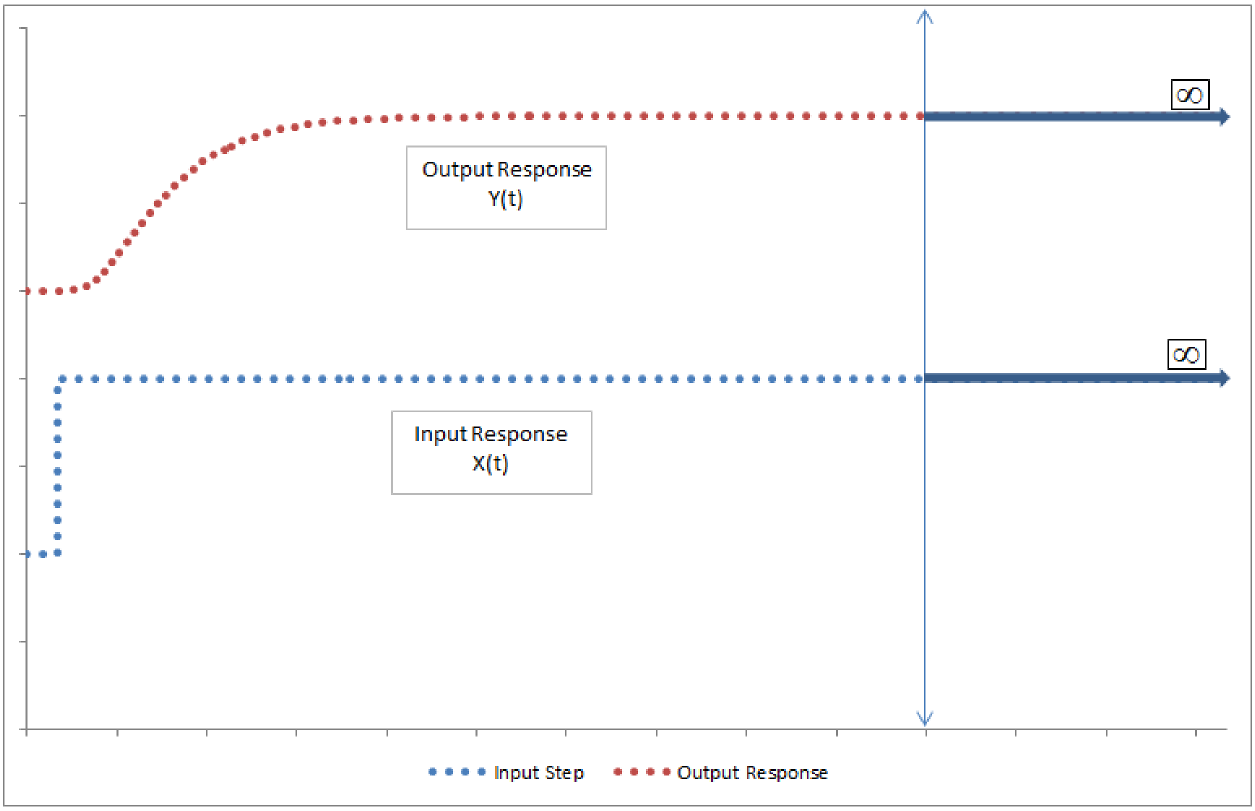

Figure 1-2 Point to Point of Test Data

When a process is described as self-regulating the time domain equations X(t) and Y(t) can be written as in this case as 3001 point to point lines connecting the 3001 points from the beginning of the test to the end of the test data. The equation for a straight line from the end of the test data to infinity is then added to the real data to get two time domain equations from time zero to time infinity made up of in this case of 3001 individual equations.

Integrating the two continuous time domain equations from time zero to time infinity e^{-st} we now have a Laplace transformation of the process of P(s) which exactly describes the real transfer function of the process. The Laplace transfer function is not like any standard function typically seen or written with the leads and lags described but none the less it is exact. Though in this form not really usable for tuning controllers. The next step in the analysis is to make the data useful.

The digital Laplace transform that describes the process contains the Laplace operator s. In fact it contains thousands of s's.

Equation 1-3 Substituting iω for the Laplace Operator s in Process Transfer Function

$$P(s) = \left|\frac{Y(s)}{X(s)}\right| s = i\omega$$

$$i = \sqrt{-1}$$

$$\omega = \text{Angular Frequency (rad/sec)}$$

At this point we now have a process transfer function where the only unknown is the angular frequency ω. If we now separate all the real and imaginary components we have transfer function in the format.

Equation 1-4 Dynamic Gain and Phase Shift as Function of Angular Frequency ω

$$Process = Real + (i)\,Imaginary$$

$$Dynamaic\,Gain = \sqrt{Real^2 + Imaginary^2}$$

$$Phase\ Shift\theta = arctangent\,\frac{Imaginary}{Real}$$

$$As\ a\ \ function\,of\ \ \omega$$

The Process Transfer Function in terms of Dynamic Gain and Phase Shift can now be plotted as a function of Angular Frequency in the well-known Bode, Nichols and Nyquist plot formats.

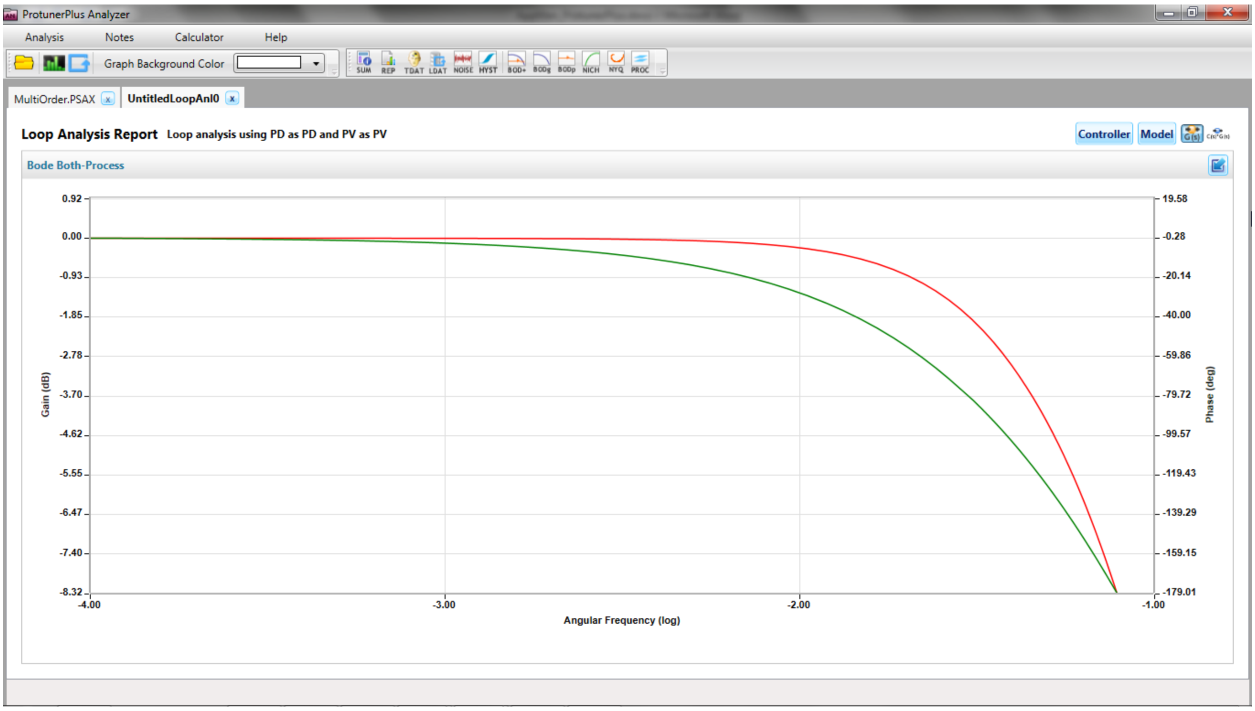

Figure 1-3 Bode Plot of Multi-Order Process from Time Domain Data in Figure 1-1

The question of how well the Bode Plot of the Process as found with the digital Laplace transformation matches the Bode Plot of the actual fifth-order equation can be found by clicking on the Model tab and entering the real model time constants and over laying the Bode Plots.

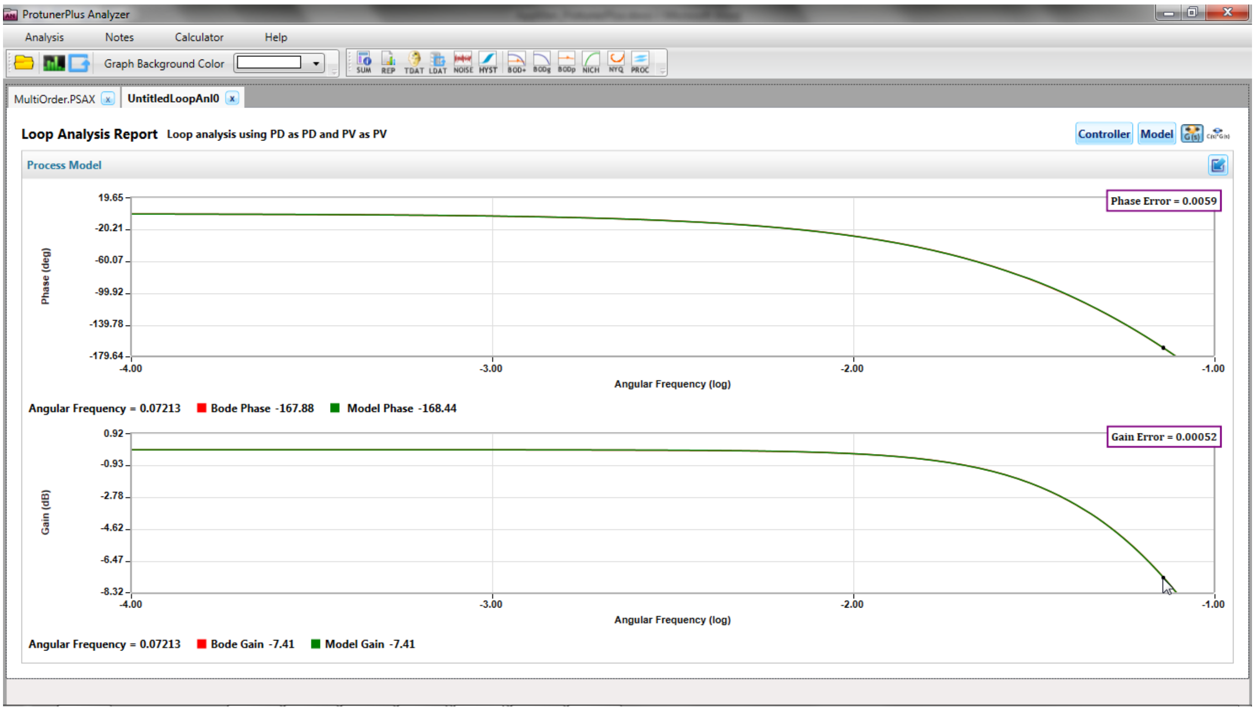

Figure 1-4 Bode Plot From Digital Laplace Transform Overlaid with Actual Model Data

The single curser point shown at an angular frequency of 0.07123 radians per second found the model gain (M) and the process gain (P) both to be exactly -7.41 db. The phase shift of the process (P) was found to -167.88 degrees vs. the model (M) phase shift of 168.44 degrees. The model and process phase shift is not exact but very close. The slight difference in the phase shift between the process and the model is a function of sample rate at which the data is taken. For all practical purposes the digital Laplace transformation of time domain data is an exact solution of the real test data.